Solve the Following Questions:

Question 1: (14 Marks)

a) Let X have a Poisson distribution with

$$P(x)=\frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x=0,1,2,\cdots$$

Find the moment generating function of the random variable X, and deduce the mean and variance.

b) Let A and B be two events with P(A)=0.3, and P(B)= 0.4

Find P(A|B) in the following cases

i- A and B disjoint

ii- A and B independent.

Question 2: (14 Marks)

Box 1 contains 2 red balls and 3 blue balls. Box 2 contains 3 blue balls and 5 red balls. Box 3 contains 4 blue balls and 3 red balls. A ball selected at random.

i) What is the probability that its red ball?

ii) If the ball selected red, what is the probability that it came's from box 3?

Question 3: (14 Marks)

Let X be a random variable with

$$f(x)=(\frac{1}{2})^{x},\ x=0,1,2,3$$

a) Is $f(x)$ a probability distribution? Satisfy.

b) Find $P(X>1)$

c) Find $P(2<X<6)$

d) Find E(X) and Var(X)

Question 4: (14 Marks)

Suppose 15 percent of total production of a given type of electric generators is defective. What is the probability that when five generators are picked randomly from this total the following are found:

a) No defectives

b) Only two defectives

c) No more than two defectives

d) Only three defectives

e) No more than three defectives

Question 5: [14 Marks]

a) The line width for semiconductor manufacturing is assumed to be normally distributed with mean 0.5 micrometer and standard deviation of 0.05 micrometer.

i) What is the probability that a line width is greater than 0.62 micrometer?

ii) What is the probability that a line width is between 0.47 and 0.63 micrometer?

b) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

i) What is the probability that a product attains a good review?

ii) If a new design attains a good review, what is the probability that it will be a highly successful product?

Standard Normal Distribution Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0190	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2624	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2969	.2995	.3023	.3051	.3078	.3106	.3313
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4926	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Good luck

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جامعة القاهرة
معهد الدراسات والبحوث الإحصائية

المادة : Stat M 507
الزمن: ثلاث ساعات

اختبار الفصل الدراسي الثاني – العام الجامعي 2014 -2015

Answer the following questions:

(1) a) How many ways can you permute 3 *a*'s, 4 *b*'s, and 2 *c*'s?

b) One urn contains one black ball and one gold ball. A second urn contains one white and one gold ball. One ball is selected at random from each urn.

(i) Exhibit a sample space for this experiment.

(ii) What is the probability that both balls will be of the same color?

(iii) What is the probability that one ball will be green?

(2) a) If $P(A) = {}^{1}/_{3}$ and $P(B') = {}^{1}/_{4}$, can A and B be mutually exclusive events? Explain.

b) A bowl contains four lottery tickets with the numbers 111, 221, 212, and 122. One ticket is drawn at random from the bowl, and A_i is the event " 2 in the i^{th} place"; i =1, 2, 3. Determine whether A_1, A_2, and A_3 are independent.

(3) a) If $P(B) = P(A|B) = P(C|AB) = 0.5$, what is $P(ABC)$?

b) Given that $P(A) > 0$ and $P(B) > 0$, prove:

If $P(A|B) = P(B|A)$, then $P(A) = P(B)$.

من فضلك: أنظر الصفحة الثانية

(4) a) Let B_1, B_2, and B_3 constitute a partition of a sample space. If $P(B_j) = 1/3$ and $P(A|B_j) = j/6$ for $j = 1, 2, 3$, what is $P(A)$?

b) Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time. (A is used the other 70%.) A worker was taught the skill by one of the methods but failed to learn it correctly. What is the probability that she was taught by method A?

(5) a) A discrete random variable has pmf $f(x)$.

(i) If $f(x) = k(1/2)^x$ for $x = 1, 2, 3,$ find k.

(ii) Is a function of the form

$f(x) = k[(1/2)^x - 1/2]$ for $x = 0, 1, 2$ a pmf for any k? Explain.

b) A continuous random variable X that can assume values between $x = 1$ and $x = 3$ has a density function given by $f(x) = 1/2$.

(i) Show that the area under the curve is equal to 1.

(ii) Find $F(x)$ and use it to evaluate $P(2 < X \leq 2.5)$.

(6) a) Let X be a random variable having a binomial distribution with parameters $n = 25$ and $p = 0.2$. Evaluate $P(X < \mu - 2\sigma)$.

b) A warehouse contains ten printing machines, four of which are defective. A company selects five of the machines at random.

من فضلك: أنظر الصفحة الثالثة