Cairo University-Institute of Statistical Studies and Research
Department of Mathematical Statistics
Degree : Diploma in Statistics Course Name : StatM512
Final Exam. January 2009 Time allowed: 3 hours

Answer the Following Questions:

1. Let X be $N(\theta,\sigma^2)$, where σ^2 is known , and the prior density of θ be $N(\mu_0,\sigma_0^2)$, obtain the posterior density of θ .

2- Let $x_1, x_2 ....x_n$ be a random sample of size n from the following distribution

$$f(x)=\begin{cases}\frac{1}{\theta} & ,0<x<\theta\ ,\theta>0\\ 0 & otherwise\end{cases}$$

Obtain posterior distribution of θ (use prior based on Fisher Information) and then obtain Bayesian point and interval estimator for θ .

3-Suppose that the posterior distribution of θ is given by

$$\pi(\theta\mid data)\ \alpha\ \theta^{-0.5(n+\nu_0)-1}\ \exp(-(\nu_0\sigma_0^2+S^2)/\theta)$$

Where n is the sapmle size , μ is the population mean , ν_0,σ_0^2 are prior parameters and

$$S^2=\sum_{i=1}^{n}(x_i-\mu)^2$$

Derive shortest confidence interval for θ.

4-Suppose $x_1, x_2 ....x_n$ form a random sample from $N(\mu,1)$ and the prior distribution of μ is uniform on the interval $(-\infty,\infty)$. Use the following data

16.0 17.0 16.5 15.2 18.0 18.2 15.5 16.4 16.6 15.0

, $\alpha=0.05$ and Bayesian approach to test the following hypotheses

$$H_0:\mu=17 \quad versus \quad H_1:\mu\neq 17$$