



Answer the following questions:

(1) If X_1, X_2, \dots, X_n are IID from $uniform(0, 1)$:

(a) Show that $X_{(j)}$ has a $Beta(j, n - j + 1)$ distribution.

(b) Find the conditional PDF of $X_{(1)}$ given $X_{(n)}$.

(2) Let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ denote the order statistics of a continuous random variable with PDF $f(x)$ and CDF $F(x)$. Obtain the PDF of $U = F(x_{(s)}) - F(x_{(k)})$.

(3) Let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ denote the order statistics of a random sample of size n from a probability distribution having:

$$f(x) = \text{Exp}(-x), x > 0.$$

Show that the statistics $Z_1 = X_{(k)}$ and $Z_2 = X_{(s)} - X_{(k)}$ are statistically independent. ($s > k$)

(4) For the $uniform(0, 1)$ distribution, the random variables

$$V_1 = \frac{X_{(k)}}{X_{(s)}} \text{ and } V_2 = X_{(k)}, 1 \leq k < s \leq n$$

Are statistically independent, what is the distribution of V_1 .