Subject: Statistical Inference II	Subject code: Ms524	Time: 3 Hours	Exam marks: 75	# Exam. Sheets: 1

Examiner : Dr. Hiba Zeyada

Question One:

a) Neyman –Pearson lemma states that:

" If C is a critical region of size α and K is a constant such that $\frac{L_0}{L_1} \leq K$ inside C and $\frac{L_0}{L_1} \geq K$ outside C, then C is the most powerful critical region for testing $\theta = \theta_0$ against $\theta = \theta_1$". **Prove the above lemma**

b) Given a random sample of size n=20 from $N(\mu, 5)$ use Neyman –Pearson lemma to find the MPT of size $\alpha = 0.05$ to test $H_0: \mu = 7$ versus $H_1: \mu > 7$.

Question Two:

a) Suppose we wish to test $H_0: \mu = 15$ versus $H_1: \mu = 16$ with $\alpha = \beta = 0.05$. Find the sample size that will ensure this accuracy assume $\sigma^2 = 9$.

b) Given a random sample of size n from a normal distribution with mean μ (unknown) and variance σ^2. For testing $H_0: \sigma^2 = \sigma_0^2$ versus $H_1: \sigma^2 > \sigma_0^2$, show that the likelihood ratio test is equivalent to the chi- square test.

Question Three: True or false? Correct the false

1- The random variable $\frac{(n-1)S^2}{\sigma^2}$ is distributed as chi-square with n-1 degrees of freedom

2- The p-value of a test is the largest value of α that would lead to accept the alternative hypothesis.

3- The random variable $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ is standard normal distributed.

4- The power function is the probability to accept the null hypothesis

5- If **X** is a continuous random variable then the distribution of $F_X(x)$ is **uniform** with parameters **a** and **b**

6- A type I error is made if H_0 is rejected when H_1 is true

7- A Test is MPT of size α if it has the size of its type I error equal to α and has smallest type II error among all other tests with size of type I error α or less.

Hint: use $Z_{0.05} = 1.64$

Cairo University
Institute of Statistical Studies & Research
Course Name: Statistical Inference(2)
Course code: STATM524
Instructors: Prof. Shaban Abdel-hameed
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Year:2011-2012 (Spring semester)
Final Exam
Time allowed: 3 hrs
Marks 70

Solve the Following Questions

Question one: (15 Marks)

a) State and prove Nayman -Pearson theorem for the best critical region of size α for testing the simple hypothesis $H_0 : \theta = \theta'$ against the alternative simple hypothesis $H_a : \theta = \theta''$

b) Suppose that $x_1, x_2, \cdots, x_n$ denote a random sample from a population having an exponential distribution with density $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $x > 0$ Derive a most powerful test for testing $H_0 : \theta = \theta_0$ vs $H_a : \theta = \theta_a$ when $\theta_a < \theta_0$.

Question two: (12 Marks)

Suppose that $y_1, y_2, \cdots, y_n$ denote a random sample from a population having a Poisson distribution with probability distribution $f(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0,1,2,\cdots$ find the form of the rejection region (R.R) for a most powerful test for testing $H_0 : \lambda = \lambda_0$ vs $H_a : \lambda = \lambda_a$ when $\lambda_a > \lambda_0$.

Question three: (13 marks)

The number of computer malfunctions per day is recorded for 260 days. Test number of malfunctions follows Poisson distribution with parameter $\lambda = 1.15$

Number of malfunctions(x_i)	0	1	2	3	4	5
Number of days f_i	77	90	55	30	5	3

Question four: (15 marks)

Let X has a normal distribution with mean μ and variance $\sigma^2 = 100, n = 64$ and $RR = \{(x_1, x_2, \cdots, x_n) \mid \bar{x} < 98 \text{ or } \bar{x} > 102\}$ and the test is $H_0 : \mu = 100$ vs $H_a : \mu \neq 100$

Find:

i) Type I error. ii) Type II error when $\mu = 101$ iii) power of the test

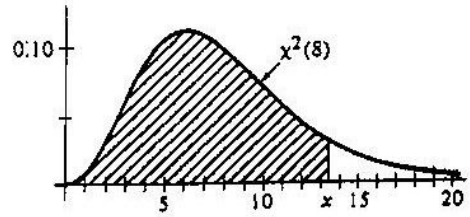
Question five: (15 marks)

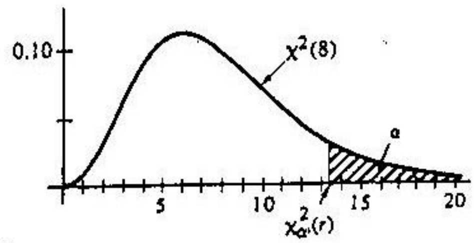
Let $x_1, x_2, \cdots, x_n$ be iid random sample distributed gamma with density defined by

$f(x) = \frac{x}{\theta^2} e^{-\frac{x}{\theta}}$, $x > 0$

a) Verify that $\hat{\theta}$ is an unbiased estimator of θ.
b) Compute the Cramer Rao lower bound for $\hat{\theta}$.
c) Show that $\hat{\theta}$ is a minimum variance unbiased estimator.
d) Is $\hat{\theta}$ complete sufficient statistics.

TABLE IV
The Chi-Square Distribution



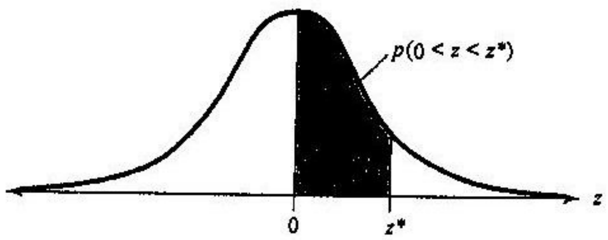


$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2}\, dw$$

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58
16	5.812	6.908	7.962	9.312	23.54	26.30	28.84	32.00
17	6.408	7.564	8.672	10.08	24.77	27.59	30.19	33.41
18	7.015	8.231	9.390	10.86	25.99	28.87	31.53	34.80
19	7.633	8.907	10.12	11.65	27.20	30.14	32.85	36.19
20	8.260	9.591	10.85	12.44	28.41	31.41	34.17	37.57
21	8.897	10.28	11.59	13.24	29.62	32.67	35.48	38.93
22	9.542	10.98	12.34	14.04	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96
28	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
40	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69
50	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
60	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
70	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4
80	53.34	57.15	60.39	64.28	96.58	101.9	106.6	112.3

This table is abridged and adapted from Table III in *Biometrika Tables for Statisticians*, edited by E. S. Pearson and H. O. Hartley. It is published here with the kind permission of the *Biometrika* Trustees.

F Body Table for the Standard Normal Distribution



z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4692	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990