Cairo University Institute of Statistical Studies and Research First Term 2010-2011 Final Exam	(Stat M 527) Time : 3 Hours

Answer the following questions :

Q1 : : Let X and Y be continuous random variables with joint probability density function given by $f(x, y) = e^{-(x+y)}$ $x, y > 0$. Find

(i) The marginal density functions for X and Y .

(ii) P(X > Y). (iii) P(X + Y > 1)

(iv) Are X and Y are independent.

Q2 : (i) If the random variable X has probability density function

$$f_X(x) = \begin{cases} \dfrac{kx^{p-1}}{(1+x)^{p+q}} & x \geq 0; \quad p, q > 0 \\ 0, & otherwise, \end{cases}$$

Find the probability density function of $Y = \frac{1}{X+1}$.

(ii) If $X \sim \beta(\alpha, \theta)$ **and** $Y \sim \beta(\alpha + \theta, r)$ and X , Y are independent . Find the distribution of XY.

Q3 : Find the probability density function of $Y = X^2$ when the probability density function of X is given by

(i) $f_X(x) = x, \quad -1/2 \leq x \leq 3/2$

(ii) Suppose $Z \sim N(0,1)$. Show that $V = Z^2$ has the χ^2 distribution with one degree of freedom.

Q4: Let the joint probability density function of the random variables (X, Y) is

$$f(x, y) = K(1-x)^{\alpha} y^{\beta}, \quad k = \frac{(\beta+1)}{B(\beta+2, \alpha+1)} \quad and \quad 0 \leq y \leq x \leq 1; \ \alpha, \beta > -1.$$

(i) Calculate $g(x)$. Find the conditional probability density function $f(y/x)$ and hence Find $E(Y/x)$.

(ii) Calculate the marginal probability density function of Y, $h(y)$, and the conditional probability density function $f(x/y)$. Indicate how to calculate $E(X/y)$.

Q5: If the random variable X is a standard normal distribution and Y is an independent distributed chi-square with n degree of freedom. Find the distribution of the random variable $\mathbf{Z} = \dfrac{\mathbf{X}}{\sqrt{\dfrac{\mathbf{Y}}{\mathbf{n}}}}$

$X \sim \beta(a,b) \qquad f(x) = \dfrac{1}{\beta(a,b)} \; x^{a-1} \; (1-x)^{b-1} \;, \; 0 < x < 1$

$X \sim \chi_n^2 \qquad f(x) = \dfrac{1}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} \; x^{\frac{n}{2}-1} \; e^{-\frac{x}{2}}$

$X \sim \text{Gamma}(a,b) \qquad f(x) = \dfrac{b}{\Gamma(a)}(bx)^{a-1} e^{-bx}$

$X \sim t(n) \qquad f(x) = \dfrac{1}{\sqrt{n}\beta(\frac{n}{2},\frac{1}{2})}(1+\dfrac{x^2}{n})^{-(\frac{n+1}{2})}$

$X \sim F(m,n) \qquad f(x) = \dfrac{(\frac{m}{n})^{\frac{m}{2}}}{\beta(\frac{n}{2},\frac{1}{2})} \; X^{\frac{m}{2}-1}(1+\dfrac{m}{n}x)^{-(\frac{m+n}{2})}$

Cairo University
Institute of Statistical Studies & Research
Course Name: Probability& Statistics(2)
Course code: STATM527
Instructor: Dr. Elsayed Elsherpieny

Year:2011-2012 (Spring semester)
Final Exam
Time allowed: 3 hrs
Marks 70

Solve the Following Questions: (Marks)

Question one: (30 Marks)

Show that the following function satisfies the properties of a joint probability mass function.

X / Y	-1	-0.5	0.5	1
-2	1/8	0	0	0
-1	0	1/4	0	0
1	0	0	1/2	0
2	0	0	0	1/8

Determine the following:

a) $P(X < 0.5, Y < 1.5)$ b) $P(X < 0.5)$

c) $P(X > 0.25, Y < 4.5)$ d) $P(Y < 1.5)$

e) $E(X), E(Y), V(X)$, and $V(Y)$

f) Marginal probability distribution of X given that Y=1

g) Conditional probability distribution of X given that Y=1

h) E(X|y=1)

i) are X and Y independent?

j) Determine the covariance and correlation

Question two: (10 Marks)

A random variable X has the following probability distribution

$$f(x) = e^{-x} \qquad , x > 0$$

i) find the probability distribution function $Y = X^2$

ii) find the probability distribution function $Y = \sqrt{X}$

iii) find the probability distribution function $Y = \ln X$

Question three: (10 marks)

Let X_i be random variable has a gamma distribution with probability density function as follows:

$$f(x_i) = \frac{1}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-x_i} \qquad , x_i > 0$$

Find the marginal distribution of $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1 + X_2}$

Show that Y_1 and Y_2 are independent

Question four: (10 marks)

Let X has a normal distribution with mean μ and variance $\sigma^2 (-\infty < \mu < \infty, \sigma > 0)$

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty$$

Show that the moment generating function $M_X(t) = \exp\{\mu t + \frac{1}{2}\sigma^2 t^2\}$, **deduce the mean and variance**

Question five: (20 marks)

Determine the value of c that makes the function $f(x, y) = c(x + y)$ **a joint probability density function over the range** $0 < x < 3$ **and** $x < y < x + 2$

Determine the following:

2 a) $P(X < 1, Y < 2)$

2 b) $P(1 < X < 2)$

2 c) $P(Y > 1)$

2 d) $E(X), E(Y), V(X)$, and $V(Y)$

2 e) Marginal probability distribution of Y given that X=1

2 f) E(Y|x=1)

2 g) P(Y>2|x=1)

2 h) are X and Y independent?

Good luck

Dr. Sayed Elsherpieny