Cairo University

Institute of Statistical Studies & Research

Course Name: Stochastic Processes in Demo.

Instructor: Prof. Elsayed Elsherbeny

Year:2014-2015(Spring semester)

Course code: StatM 602

Time allowed: 3 hrs

Marks 70

Solve the following questions:

Question one:[15 marks]:

a) Define: regular stochastic matrices – periodic state – absorbing state.

b) For the transition matrix

$$T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$\frac{1}{3} - \lambda$

i) Find the eign values and corresponding eign vectors

ii) Calculate $T_{12}^{(3)}, T_2^{(3)}$ and $T^{(3)}$ given that $T^{(0)} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

Question two [20 marks]:

a) Define: Ergodic Chains- Closed sets- Irreducible Chain.

b) Given the following matrix:

$$T = \begin{bmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

i. Is T stochastic matrix?

ii. Is T regular?

iii. Find the unique fixed probability vector of T?

iv. Is the state E_3 absorbing state?

v. What matrix does T^n approach?

vi. What vector does $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} T^n$ approach?

vii. Find and interpret T_{23}^3, T^3, *and* T_2^3

Question three [20 marks]

The weather in a certain region can be characterized as being sunny (S), cloudy (C) or rainy (R) on any particular day. The probability of any type of weather on one day depends only on the state of the weather on the previous day. For example, if it is sunny one day then sun or clouds are equally likely on the next day with no possibility of rain. Explain what other the day-to-day possibilities are if the weather is represented by the transition matrix

i) Construct the transition probability matrix T .

ii) Find the formula for T^n .

iii) In the long run, if the weather is cloudy, what percentages of the days are sunny, cloudy and rainy?

Question four [15 marks]

a- Define: Persistent state – Null state – Transient state.

b- A three state Markov chain has the transition matrix

$$T_n = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{n+1} & 0 & \frac{n}{n+1} \end{bmatrix}$$

Where T_n is the transition matrix at step n. Show that the state E_1 is persistent null state?

Good luck
Prof. Elsayed Elsherbeny

1) Let X be a random variable with cumulative distribution function

$$G(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x}{4} e^{-x/4} & \text{for } 0 \le x \end{cases}$$

a) Give the probability density function for X
b) Find $\Pr\{X \ge 4\}$
c) Find $\Pr\{4 < X \le 8\}$

2) Let U be a random variable with pdf given by

$$h(u) = \begin{cases} 0 & \text{for } u < 0 \\ u & \text{for } 0 \le u < 1 \\ 2 - u & \text{for } 1 \le u < 2 \\ 0 & \text{for } u \ge 2 \end{cases}$$

Find the mean and standard deviation of U.

3) Let S and T be two continuous random variable with joint pdf given by

$$g(s,t) = \begin{cases} e^{-\frac{1}{2}s - 2t} & \text{for } s \ge 0, t \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Find the marginal pdf's for S and T and then find the $\Pr\{T \le 1\}$ and $E(T)$
b) Find the conditional pdf for S given that $T = 1$ and then find $P\{S \le 1 | T = 1\}$ and find $E\{S | T = 1\}$

4- Let X be a Markov chain with state space {a, b, c} and transition probabilities given by

$$P = \begin{bmatrix} 0.3 & 0.7 & 0.0 \\ 0.0 & 0.6 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

Let the initial probabilities be given by the vector (0.1, 0.3, 0.6). Find

(a) $\Pr\{X_2 = b \mid X_1 = c\}$
(b) $\Pr\{X_3 = b \mid X_1 = c, X_0 = c\}$
(c) $\lim_{n\to\infty} \Pr\{X_n = b \mid X_0 = a\}$