Solve the following questions:

Question one: (14 marks)

a) Consider the two-state chain with transition matrix

$$T = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}, \quad 0 < \alpha, \beta < 1.$$

i) **Find the eignvalues and corresponding eignvectors.**
ii) **Construct a formula for T^n**
iii) **Which T^n tends as $n \to \infty$**

b) For the transition matrix

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Calculate $P_{12}^{(2)}, P_2^{(2)}$ and $P^{(2)}$ given that $P^{(0)} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Question two: (14 marks)

a- Define: Regular transition matrix - Absorbing state

b- Suppose that T is the transition matrix of three state Markov chain,

$$T = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

i) **Is T a regular stochastic matrix.**
ii) **Find the unique fixed probability vector.**
iii) **What matrix does T^n approach?**
iv) **What vector does $[1 \quad 0 \quad 0]T^n$ approach?**
v) **Comment on the results.**

Question three: (14 marks)

2) a- Define: Periodic state- Persistent state .

b- A three state Markov chain has the transition matrix

$$T_n = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{n+1} & 0 & \frac{n}{n+1} \end{bmatrix}$$

Where T_n is the transition matrix at step n. Show that the state E_1 is persistent null state.

Question four: (14 marks)

a- Define: Ergodic chains - Closed sets - Irreducible chain.

b- There are 2 white balls in box A and 3 red balls in box B. At each step in the process, a ball is selected from each box and the two balls are interchanged. The system may be described by three states, which denotes the numbers of red balls in box A.

i) Find the transition probability matrix P .

ii) Find the probability that there are 2 red balls in box A after 3 steps.

iii) Find the probability that, in the long run, there are 2 red balls in box A.

Question five: (14 marks)

a) **Null state – Transient state.**

b) **Two boys b_1 and b_2 and two girls g_1 and g_2 are throwing a ball from one to the other. Each boy throws the ball to the other boy with probability $\frac{1}{2}$ and to each girl with probability $\frac{1}{4}$. On the other hand, each girl throws the ball to each boy with probability $\frac{1}{2}$ and never to the other girl. In the long run, how often does each receive the ball?**

With my best wishes
Dr. ElsayedElsherpieny

Cairo University
Institute of Statistical Studies & Research
Course Name: Stochastic Processes
Course Code: STATM605
Instructor: Prof. Elham Shoukry Dr. Elsayed Elsherpieny

Year:2012-2013
Final Exam 1^{ST} term
Time allowed: 3 hrs
Marks 70

Solve the following questions:

Question one: (14 marks)

a) Consider the two-state chain with transition matrix

$$T = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}, \quad 0<\alpha,\beta<1.$$

i) **Find the eigenvalues and corresponding eigenvectors.**

ii) **Construct a formula for T^n**

iii) **Which T^n tends as $n \to \infty$**

b) For the transition matrix

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Calculate P_{12}^2, and $P_2^{(2)}$ given that $P^{(0)} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

Question two: (14 marks)

a- **Define: Regular transition matrix - Absorbing state**

b- **The weather in a certain region can be characterized as being sunny (S), cloudy (C) or rainy (R) on any particular day. The probability of any type of weather on one day depends only on the state of the weather on the previous day. Explain what other the day – to - day possibilities are if the weather is represented by the transition matrix**

$$T = \begin{bmatrix} & S & C & R \\ S & \frac{1}{2} & \frac{1}{2} & 0 \\ C & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ R & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Find the eigenvalues of T. In the long run, what percentages of the days are sunny, cloudy and rainy?

Question three: (14 marks)

a) **Define: Null state – Transient state.**

b) **Suppose that T is the transition matrix of three state Markov chain,**

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

i) **Is T a regular stochastic matrix.**

ii) **Find the unique fixed probability vector.**

iii) **What matrix does T^n approach?**

iv) **What vector does $[1 \quad 0 \quad 0]T^n$ approach?**

v) **Comment on the results.**

Question four: (14 marks)

a- Define: Periodic state- Persistent state .

b- A three state Markov chain has the transition matrix

$$T_n = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{n+1} & 0 & \frac{n}{n+1} \end{bmatrix}$$

Where T_n is the transition matrix at step n. Show that the state E_1 is persistent null state.

Question five: (14 marks)

a- Define: Ergodic chains - Closed sets - Irreducible chain.

b- There are 2 white balls in box A and 3 red balls in box B. At each step in the process, a ball is selected from each box and the two balls are interchanged. The system may be described by three states, which denotes the numbers of red balls in box A.

i) Find the transition probability matrix P .

ii) Find the probability that there are 2 red balls in box A after 3 steps.

iii) Find the probability that, in the long run, there are 2 red balls in box A.

With my best wishes

Dr. Elsayed Elsherpieny

Cairo University Institute of Statistical Studies& Research Qualified Master	Stochastic processes exam.(STATM605) Time: 3 hours Jan- 2010

Solve the following questions:

1) a- Define: **Regular transition matrix- Absorbing state**

b- Suppose **that T is the transition matrix of three state Markov** chain,

$$T = \begin{bmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

i) **Show that T is regular.**

ii) **Find the unique fixed probability vector.**

iii) **What matrix does T^n approach?**

iv) **What vector does $\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} T^n$ approach?**

v) **Comment on the results.**

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2) a- Define: **Periodic state- Persistent state – Null state.**

b- A three state **Markov chain has the transition matrix**

$$T_n = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \\ \frac{1}{n+1} & 0 & \frac{n}{n+1} \end{bmatrix}$$

Where T_n **is the transition matrix at step n. Show that the state E_1** is persistent null state.

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3)a- Define: **Ergodic chains- Closed sets- Irreducible chain.**

b- There are **2 white balls in box A and 3 red balls in box B. At each step in the** process, a ball **is selected from each box and the two balls are interchanged. The** system may be **described by three states, which denotes the** numbers of red balls in box A.

i) Find the **transition probability** matrix P.

ii) Find the **probability that there are 2 red balls in box A after 3** steps.

iii) Find the **probability that, in the long run, there are 2 red balls in box A.**

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4) a) **Consider the two-state chain with transition matrix**

$$T=\begin{bmatrix}1-\alpha & \alpha \\ \beta & 1-\beta\end{bmatrix},\quad 0<\alpha,\beta<1.$$

i) **Find the eignvalues and corresponding eignvectors.**

ii) **Construct a formula for T^n**

iii) **Which T^n tends as $n\to\infty$**

b) For the **transition matrix**

$$T=\begin{bmatrix}\frac{1}{3} & \frac{2}{3} \\ \frac{1}{4} & \frac{3}{4}\end{bmatrix}$$

Calculate $P_{12}^{(3)}, P_2^{(3)}$ and $P^{(3)}$ given that $P^{(0)}=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\end{bmatrix}$

With my best wishes
Dr. Elsayed Elsherpieny

Cairo University, I.S.S.R.
Master of Statistics

Stochastic Processes (I)
"Stat M 605"
Time: 3 hours

Second Term Exam
June 2015

Answer The Following Questions:-

Q(1):-

a) Define the following:

- **i.** Communicating states.
- **ii.** Irreducible Markov Chain.
- **iii.** Recurrent and transient states.

b) Find eigenvalues, eigenvectors and the formula for P^n for the following transition matrix:

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Q(2):-

a) Given a Markov Chain of states E_1, E_2, E_3, E_4 and E_5 and of its stochastic matrix P given by:

$$\boldsymbol{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- **i)** Draw a transition diagram for transition matrix P.
- **ii)** Prove that the Markov Chain is irreducible.

b) Classify the following Markov chain with 6 states:

$$P := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0.4 & 0.6 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.4 & 0.2 & 0.1 & 0 \\ 0 & 0 & 0 & 0.3 & 0.7 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0.3 & 0 & 0.7 \end{pmatrix}$$

- **i.** Deduce which states are transient.
- **ii.** What will be the period of each state?

See the next page ➡

Q(3):-

a) Five balls, 2 white ones and 3 black ones, are distributed randomly in two boxes A and B, such that A contains 2, and B contains 3 balls. At time n (where $n = 0, 1, 2, \dots$) we choose at random from each of the two boxes one ball and let the two chosen balls change boxes. In this way we get a Markov chain with 3 states: E_0 , E_1 and E_2, according to whether A contains 0, 1 or 2 black balls.

i. Find the corresponding stochastic matrix P.

ii. Prove that stochastic matrix P is regular.

b) (i) Classify the following transition probability matrix:

$$\boldsymbol{P} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

(ii) Prove that state s_4 is recurrent.

(iii) Find $P_{54}^{(8)}$.

Q(4):-

a) Define a Markov chain and give an example.

b) A psychologist makes the following assumptions concerning the behavior of mice subjected to a particular feeding schedule. For any particular trial 90% of the mice that went right on the previous experiment will go right on this trial, and 70% of those mice that went left on the previous experiment will go right on this trial. If 50% went right on the first trial, what would he predict for:

i) The second trial?

ii) The thousandth trail?

"With our best wishes"

Prof. Elham Shoukry & Dr. Salwa Mahmoud