1. If the density of the Gamma (α, λ) distribution is given by

$$f(t) = \frac{\lambda (\lambda t)^{\alpha-1} e^{-\lambda t}}{\overline{|\alpha}}, \quad t > 0 \ (\lambda > 0, \ \alpha > 0),$$

show that this family belongs to IFR if $\alpha \geq 1$ and to DFR if $0 < \alpha \leq 1$.

(15 points)

2. Show that if X is a life random variable with survival function $\overline{F}(\cdot)$, then

$$E(X) = \int_0^\infty \overline{F}(u) \, du$$

(10 points)

3. (i) Define each of the following

$X \in \{HNBUE\}$; $X \in \{NBUE\}$,

(ii) f is a PF_2 function ,

(iii) the failure rate of a random variable X,

(iv) the mean residual life of X,

(v) the mean inactivity time of X,

(vi) the reversed failure rate of X.

(12 points)

4. Prove that IFRA $\Longrightarrow$ NBU.

(15 points)

5. Prove that if $\{\bar{P}_K\}$ is such that

$$1 = \bar{P}_0 \geq \bar{P}_1 \geq \bar{P}_2 \geq \cdots \quad \text{and}$$

$$\bar{P}_{i+j} \leq \bar{P}_i \, \bar{P}_j \quad \text{for } i,j = 0,1,2,\ldots, \text{ then}$$

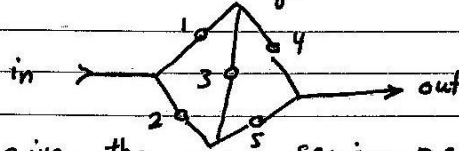$$\bar{H}(t) = \sum_{R=0}^{\infty} \frac{(\lambda t)^k e^{-\lambda t}}{k!} \bar{P}_K$$

is NBU. (15 points)

6. What needs to be established when a new family of distributions is defined? (7 points)

7. Give an example to show that the DFR family is not closed under convolutions (6 points)

8. Let ϕ be a coherent structure. Prove that
(i) $\phi(\underline{x} \amalg \underline{y}) \geq \phi(\underline{x}) \amalg \phi(\underline{y})$.
(ii) $\phi(\underline{x} \cdot \underline{y}) \leq \phi(\underline{x}) \cdot \phi(\underline{y})$.
(10 points)

9. Consider the bridge

give the series-parallel and the parallel-series representations of the bridge. (10 points)