Cairo University	Dep. of Math. Of Stat.	Final Exam
ISSR	STAT M610	2011-2012

1:- let h(t) have the general power function h(t)= βe^{α} , where β, and α are positive. Find R(t), f(t),and MTTF.

2:- Determine the MTTF of a 2-out-of-4 system with independent components each having a constant hazard of 0.000085 failures per hour.

3:- consider the case of exponential failure and repair time distributions. Derive an expression for the renewal density . what are the availability A(t) and the steady-state availability.

4:- A pump operates continuously with a mean time to fail of 200 hours that follows the exponential distribution. A second identical pump is placed in standby redundancy, and the mean time to fail while the pump is inactive is 1000 hours. The standby time to fail is also exponentially distributed. What is the mean time to fail for the system, and what is the system reliability at time 300 hours?

5:- Assume that n units are placed under test and the failure times t_i's of the failed units are recorded and reordered in an increasing order. Let T be the censoring time of the test. Thus , $t_1<t_2<t_3<\dots\dots<t_r$, the remaining n-r units are censored . assuming that the failure times follow the exponential distribution $f(t;\theta) = \theta e^{-t\theta}$, t>0 and $\theta > 0$

a) What is the point estimate of the distribution's parameter?
b) Construct a (1-α) percent confidence interval for the parameter θ.

6-: the failure time of the system follows a Weibull distribution with a p.d.f. of the form

$$f(t) = \frac{\gamma t^{\gamma-1}}{\theta^{\gamma}} \exp\left(-\left(\frac{t}{\theta}\right)^{\gamma}\right)$$

And its repair time follows an exponential distribution with a p.d.f. of $g(t) = \mu e^{-\mu t}$

Determine

a)MTBF b) MTTR

c) what is the steady-state availability of the system A, If

$\theta = 5000000, \gamma = 2.15, and\ \mu = 10000$?

7**:- Beam lead bonds of large-scale bonds integrated circuits (ICs) were observed. Two types of opens were observed on the failed ICs. The first type was a combination of silicon to beam interface separation and broken beam on the edge of the silicon chip. The second type was that of a broken beam at the heel or midspan. Assume that the failure rate of the ICs are constant with parameters λ_1 and λ_2 and the failed ICs are repaired with constant repair rates $\mu_1$and $\mu_2$for the first and second type of failures, respectively.

a) Drive an expression for the ICs reliability at time t.
b) What is the steady – state availability of an IC?

Cairo University
Institute of Statistical Studies and Research
Department of Mathematical Statistics

Final Exam First 2016
STATISTICAL THEORY OF REIABLITY
STAT M 610

70

Name: ……………………….. ID#: ………………………………..

Answer all the following questions
***Q₁*: Complete the following:**

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a- *The reliability of a series system is always ………………………………………… that of the component with lowest reliability.*

b- *If the time to failure distribution is Rayleigh distribution, then the hazard function is ……………………………………………………………………………………*

c- *Consecutive – k – out – of n : F system is…………………………………………*
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

d- *If the hazard rate function* $h(t)$ *can be expressed as*

$$h(t) = \frac{\gamma}{\theta} t^{\gamma-1}$$

then its pdf is given as $f(t)$ = ……………………………………………………………

which is the ……………….. *model*

e- *In general , series-parallel systems have higher reliabilities than parallel – series systems when …………………………………………………………………… and ……………………………………………………………………………………*

f- *If* $h(t)$ *is the hazard function, then we can obtain the reliability function* $R(t)$

as $R(t) =$ ……………………………………………………………………………
……………………………………………………………………………………………

g- *Steps to evaluating a system's reliability are*

1- ……………………………………………………………………………………

2- ……………………………………………………………………………………

3- ……………………………………………………………………………………

h- *The reliability of a parallel system is the probability that* ………………………
……………………………………………………………………………………………

i- *Let us consider n identical nonrepairable systems and observe the time to failure for them. Assume that the observed times to failure are* $t_1, t_2, \ldots, t_n$. *The estimated mean time to failure, MTTF^ =* ………………………………………

j- *The mean residual life function of a system , L(t), is* ……………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

Or

L(t) = ……………………………………………………………………………………

k- *If T is a random variable denoting the time to failure, then the reliability function at time t can be expressed as* $R(t) =$ ………………………………………

l- *The hazard function is defined as* ……………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

m- *If the hazard function is constant, then the distribution of time to failure will be* ……………………………………………………………………………………

n- *The cumulative hazard function* $H(t)$ *is* ……………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………
……………………………………………………………………………………………

o- *The reliability of a parallel system is* ………………………*the reliability of the most reliable unit in the system.*

Q$_2$:

A manufacturer uses rotary compressors to provide cooling liquid for a power generating unit. Experimental data show that the failure times (between 0 and 1 year) of the compressors follow a beta distribution with $\alpha = 3$ and $\beta = 2$

*What is the mean residual life (**in months**) of a compressor given that the compressor has survived **three months**?*

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(Hint $f(t) = \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\,\Gamma\beta}\, t^{\alpha-1}\,(1-t)^{\beta-1}, 0 < t < 1$)

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Q3:

The failure time of an electronic device is described by a Pearson type V distribution with probability density function given by

$$f(t) = \begin{cases} \dfrac{t^{-(\alpha+1)}\, e^{-\frac{\beta}{t}}}{\beta^{-\alpha}\ \Gamma\alpha} & ,t > 0 \\ 0 & otherwise \end{cases}$$

The shape parameter $\alpha = 4$ *and the scale parameter* $\beta = 3000$ *hours*

a- *Determine the mean time to failure MTTF of the device*

b- *If the device is specified to be warranted for 2000 hours, what should the value of the scale parameter* β *be to meet the requirement?*

Q4 :

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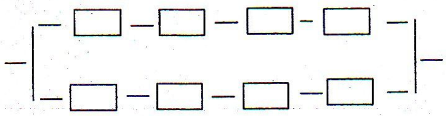
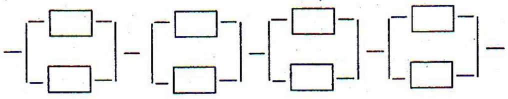
Given eight identical units each having a reliability of 0.9, determine the reliability of the following three systems resulting from the arrangements of the units in parallel-series, series – parallel and mixed – parallel configurations.

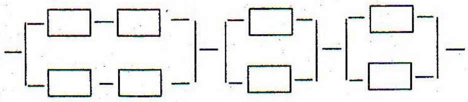
a- *Parallel – series:*



b- *Series – Parallel :*



c- *Mixed – parallel :*



Q_5 :

a- *Consider a system has five components that are connected in series. Each component has a reliability $p = 0.8$. The system fails if 3 consecutive components fail*

10

i- *What is the name of the system?*

ii- *Determine the reliability of the system*

b- *The probability density function of the failure time of the PCBs (Printed Circuit boards)to be used in a plug-compatible video display terminal is found to follow a Pareto distribution, which is given by*

$$f(t) = \lambda\, t^{-(\lambda+1)}, \qquad t \gg 1, \qquad \lambda > 0$$

i- *Find the reliability function and the hazard rate*

$R(t) =$

$h(t) =$

ii- *Is the hazard rate increasing, decreasing or constant ?*

Q6:

If the hazard rate of a brake system is found to be

$h(t) = \frac{1}{25}t^{-1/4}$ *failure* ***per year***

10

a- *What is the reliability at* $\mathbf{t = 10^4}$ ***hours****?*

b- *If 200 systems are subjected to a test at the same time, how many systems would have survived in* ***2 years*** *of operation?*

c- *What is the expected number of failures in* ***one year of operation****?*

Good luck

Cairo University	Stat M610	Finale Exam
ISSR	three hours	Second Term 2008

1:- Suppose that an object obeys an **exponential failure law** with $\lambda = 2$ per month.

(a) What is its expected lifetime?
(b) What is its hazard rate at age 1 mo.?
(c) What is the probability that it will not fail within the first month? Within the first two months?
(d) Given that it does not fail within the first month, what is the probability that it will fail during the second month?

2:- Find f(t), R(t)and MTTF , assuming

$$h(t)=\frac{\gamma}{\theta}t^{\gamma-1}$$

determine the conditions that make the hazard function increasing, decreasing , or constant.

3:- A system consists of three components with hazard rates

$h_1(t)=\lambda_1$, $h_2(t)=\lambda_2 t$ and $h_3(t)=\lambda_3 t^{\theta}$. determine the reliability and MTTF, assuming that the three components are connected in

a) Series
b) Parallel
c) Components 1and 2 are connected in parallel while component 3 is connected in series with them.

4:- A manufacturer uses rotary compressor to provide cooling liquid for a power generating unit. Experimental data show that the failure times (between 0 and 1 year) of the compressors follow a beta distribution with $\alpha=4$ and $\beta=2$. What is the mean residual life of a compressor given that the compressor has survived five months?

5:- assume n units are subjected to a test and r different failure times are recorded as $t_1<t_2<\ldots\ldots<t_r$. The remaining n-r units are censored

and their censoring times are $t_r = t_1^+ = t_2^+ = \ldots\ldots = t_{n-r}^+$. Assuming that the failure times follow the special Erlang distribution, whose P.d.f. is

$$f(t) = \frac{t}{\theta^2} e^{-\frac{t}{\theta}} \quad , \; t>0$$

estimate the distribution's parameter.