Q1): If $Y = (y_1 \quad y_2 \quad y_3)'$ is a random vector such that:

$$E(Y) = \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}, \quad \text{cov}(Y) = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{pmatrix}$$

Let $Z = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} -15 \\ -18 \end{pmatrix}$

i) Determine $E(Z)$, $\text{cov}(Z)$, $E(ZZ')$

ii) Determine the mean and the variance of z_1

iii) Determine the covariance between z_1 and $z_2 + z_3$

iv) Find the expectation of $Q = Y'AY$ where $A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

Q2): Suppose (y_{ij}) is a collection of independent random variables having a common unknown variance σ^2 and expectation:

$$E(y_{ijk}) = \mu + \beta_i + \beta_j \quad \text{where } 1 \le i \le j \le 3, \ k = 1, 2$$

And $\beta = (\mu, \beta_1, \beta_2, \beta_3)$ is completely unknown.

Answer the following:

a) Determine dim Ω.

b) Determine an (n x m) matrix U such that: $E(Y) = U\varphi$, $\varphi \in R^m$ is a parameterization for E(Y).

c) For each statement below that constitutes a valid linear hypothesis on β, write out an equivalent linear hypothesis on φ for your parameterization in part (b).

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$, $H_0 : \beta_1 - 2\beta_2 - \beta_3 = 0$

$H_0 : 2\beta_1 + \beta_3 = 0$.

i

Q3): Suppose ~~...~~ _____ _____ _____ .that:

$$E(y_{ij}) = \mu + \alpha_i \quad i = 1,\ldots,3 \quad j = 1,\ldots,n_i, \ n_1 = 3, \ n_2 = 1, \ n_3 = 2$$

a) Write the model in the matrix form and determine dim Ω

b) Which of the following parametric function can be estimated

   i) $\mu + \alpha_1$,   ii) $\alpha_1 + \alpha_2 - 2\alpha_3$,   iii) α_1 ,   iv) $\alpha_1 + \alpha_2$

c) For one of the estimable parametric function in part (c), find its estimation and its BLUE.

e) Is the following function is testable? Write the formula of the test.

   i)$H_o : \alpha_1 = \alpha_3$ ii) $H_o : \alpha_2 = 2\alpha_3$

Q4): Assume $Y = (y_1,\ldots, y_4)'$ are independent and normally distributed random variables having a common unknown variance σ^2 and expectation:

$$E(y_1) = 4\beta_1 - \beta_2 \qquad\qquad E(y_2) = 3\beta_1 - \beta_2$$
$$E(y_3) = 4\beta_1 - \beta_2 \qquad\qquad E(y_4) = \beta_1$$

Where $\beta = (\beta_1, \beta_2)$ is a unknown parameter vector satisfying $\beta_2 - 2\beta_1 = 0$.

Let, $\hat{\beta}$ be gm for β , let D be such that cov ($\hat{\beta}$) = $\sigma^2 D$, and let X, Δ' be defined in the usual way. Do the following:

a) Write out X , Δ' .   ,   b) Show that β is an estimable parametric vector.

c) Is $\hat{\beta}$ unique? Why?   ,   d) Determine m = dim Ω.

e) Determine D.   ,   f) Determine cov ($\hat{\beta}_1 - 3\hat{\beta}_2$, $\hat{\beta}_1 + 2\hat{\beta}_2$).

g) For the outcome Y = (-2, 5, 4, 8)' on Y, determine estimates for β_1 and β_2 and σ^2

h) Using the outcome in part (g), place a 95% C I of $2\beta_1 - 3\beta_2$

<div dir="rtl">مع اطيب التمنيات بالنجاح</div>

2

Answer of the following questions.

1- Consider the following data

| | Column | | | |
Row	1	2	3	4
1	9,10	10,15	8,9	10,8
2	10,12	10,11	11,9	14,12
3	16,14	12,15	14,19	20,17

Analyze the above data assuming
  i) Row random and column fixed.
  ii) Both Random
  iii) Both fixed.
    In each case write the models sketch the design matrix and Ω,
    EMSE, do all proper tests. Write the sum square using projection notations.

(27 Markets)

2- Consider thee factor factorial experiment with a level of factor A,b level of factor B,c level of factor C all effects are random, assume all interaction exists. Write the model and EMSE test $\sigma_c^2 = 0$

(16 Markets)

3- Consider the following model

$E(y_1) = \theta_1$, $E(y_2) = \theta_1 + \theta_2$, $Ey_3 = 2\theta_1 - 2\theta_2$, $E(y_4) = 3\theta_1 + \theta_2$.

$y' = (3, 4, -1, 2)$.

i) Find the class of all unbiased estimators of θ_1 . is the BLUE of θ_1 in the class ?
   Check .

ii) Write X and θ and show that θ is estimable, find $\hat{\theta}$.

iii) Consider $2\theta_1 - \theta_2 = 0$

Write the model in matrix notation , show X, Δ, Ω.. Is θ estimable parametric vector, is $\hat{\theta}$ is uniquely defined? Why ? Determine $cov(\hat{\theta}_1 - \hat{\theta}_2 , 2\hat{\theta}_1 + \hat{\theta}_2)$ Determine the correlation between $\hat{\theta}_1$ and $2\hat{\theta}_1 + \hat{\theta}_2$ Determine estimates for θ_1, θ_2 and σ^2. Place 95% C.I on $3\theta_1 - \theta_2$

(27 Markets)