Cairo University

Institute of Statistical Studies and Research

Second Semester 2011-2012

Pre-Master Students

Queueing Theory (Stat M 617)

Time: Three hours

Answer The following Questions

Q1) In self service facility model $(M/M/\infty):(GD/\infty/\infty)$, prove that :

a) $P_n = \frac{\rho^n}{n!} P_0$, $n = 0,1,2,\cdots$

b) $L_s = \rho$.

Q2) Babies are born in a certain country at the rate of one birth every 15 minutes. The time between births follows an exponential distribution. Find the following:

(a) The average number of births per year.

(b) The probability that no births will occur in anyone day.

(c) The probability of issuing 55 birth certificates in 4 hours given that 40 certificates were issued during the first 2 hours of the 4-hour period.

Q3) In a general queuing model that combines both arrivals and departures based on the Poisson assumptions-that is, the interarrival and the service times follow the exponential distribution. Under the steady state condition, derive

a) the balance equations.

b) the general formula for P_n

Q4) Prove the Memoryless property: $P\{t > T + S / t > S\} = P\{t > T\}$.

Q5) Visitors' parking is limited to three spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only two cars. Other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following:

(a) The probability, P_n of n cars in the system.

(b) The effective arrival rate for cars that actually use the lot.

(c) The average number of cars in the lot.

(d) The average time a car waits for a parking space inside the lot.

Q6) Prove that the waiting time distribution for the model (M/M/1): (FCFS/ ∞/∞) is the following exponential distribution:

$$W(\tau) = (\mu - \lambda)\ell^{-(\mu-\lambda)\tau}, \quad \tau > 0.$$

Q7) In the model (M/M/R): (GD/K/K), Prove that:

a) $$P_n = \begin{cases} C_n^k \rho^n p_0, & 0 \le n \le R \\ C_n^k \dfrac{n!\ \rho^n}{R!R^{n-R}} p_0, & R \le n \le K \end{cases}$$

b) $$P_0 = \left(\sum_{n=0}^{R} C_n^k \rho^n + \sum_{n=R+1}^{K} C_n^k \frac{n!\ \rho^n}{R!R^{n-R}}\right)^{-1}$$

Cairo University

Institute of Statistical Studies and Research

Second Semester 2014-2015 **Pre-Master Students**

Queueing Theory (Stat M 617) **Time: Three hours**

Answer The following Questions

Q1) A small post office has two open windows. Customers arrive according to a Poisson distribution at the rate of 1 every 3 minutes. However, only 80% of them seek service at the windows. The service time per customer is exponential, with a mean of 5 minutes. All arriving customers form one line and access available windows on an FCFS basis.

a) What is the probability that both windows are idle

b) What is the probability that an arriving customer will wait in line?

c) Would it be possible to offer reasonable service with only one window? Explain.

Q2) In self service facility model $(M/M/\infty):(GD/\infty/\infty)$, prove that :

a) $P_n = \frac{\rho^n}{n!} P_0 \;,\; n = 0,1,2,\cdots$

b) $L_s = \rho$.

Q3) In the model (M/M/R): (GD/K/K), Prove that:

a) $$P_n = \begin{cases} C_n^k \rho^n p_0, & 0 \le n \le R \\ C_n^k \dfrac{n!\, \rho^n}{R!R^{n-R}} p_0, & R \le n \le K \end{cases}$$

b) $$P_0 = \left(\sum_{n=0}^{R} C_n^k \rho^n + \sum_{n=R+1}^{K} C_n^k \frac{n!\, \rho^n}{R!R^{n-R}}\right)^{-1}$$

Q4) In the model $(M/M/c):(GD/\infty/\infty)$, prove that $L_q = \dfrac{\rho^{c+1}}{(c-1)!(c-\rho)^2} P_0$.

Q5) Consider a car wash facility with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour, the time for washing and cleaning a car is exponential, with a mean of 10 minutes. Suppose that the facility has a total of four parking spaces. If the parking lot is full, newly arriving cars go to other facilities. The owner wishes to determine

a) The impact of the limited parking space on losing customers to the competition.

b) Expected number of empty parking spaces.

Q6) In a general queuing model that combines both arrivals and departures based on the Poisson assumptions-that is, the interarrival and the service times follow the exponential distribution. Under the steady state condition, derive

a) the balance equations.

b) the general formula for P_n

Q7) Determine the minimum number of parallel servers needed in each of the following (Poisson arrival and departure) situations to guarantee that the operation of the queuing situation will be stable (i.e., the queue length will not grow indefinitely):

a) Customers arrive every 5 minutes and are served at the rate of 10 customers per hour.

b) The average interarrival time is 2 minutes, and the average service time is 6 minutes.

My Best Wishes