Second Term Exam	**Cairo University**
M. Sc. / Statistics	**I.S.S.R**
Multivariate Analysis (2)	**(2014-2015)**
STAT M 626	**3 Hours**

Answer the Following Questions:

1- Name the distributions, with its parameter(s), of the following:

i) $(\underline{X}-\underline{\mu}_0)'\Sigma^{-1}(\underline{X}-\underline{\mu}_0)$, **where** $\underline{X}\sim N_P(\underline{\mu},\Sigma)$ **and** Σ **has rank** p.

ii) $\sum_{r=1}^{n}\underline{X}_r\underline{X}_r'$ **, where the** p **component vectors** $\underline{X}_r$; $r=1,2,\ldots n$ **are independently distributed as** $N_p(\underline{\mu}_r,\Sigma)$.

iii) $C'WC$ **, where** $W\sim W_P(n,\Sigma,M)$ **and C is any** $(p\times q)$ **matrix of constants.**

iv) $\dfrac{n-p+1}{np}T_p^2(n)$, **where** $T_p^2(n)=k(\underline{X}-\underline{\mu}_0)'S^{-1}(\underline{X}-\underline{\mu}_0)$, $\underline{X}\sim N_p(\underline{\mu},\frac{1}{k}\Sigma)$, k **is scalar,** $nS_{\underline{X}}\sim W_p(n,\Sigma_{\underline{X}})$, **and** $\underline{X}$ **&** $nS_{\underline{X}}$ **are independent with** $n>p-1$.

v) $\dfrac{n-m}{(n-1)m}F^2$,**where** $F^2=n(C'\overline{\underline{X}}-\underline{\phi})'(C'SC)^{-1}(C'\overline{\underline{X}}-\underline{\phi})$, $\underline{X}\sim N_p(\underline{\mu},\Sigma)$, n **is the sample size, S is the sample variance covariance estimate of** Σ, **C is a given matrix of order** $(p\times m)$ **say of rank** m **and** $n>m$, $\underline{\phi}$ **is a given vector of constant.**

2- Prove that: If $\underline{X}\sim N_p(\underline{\mu},\Sigma)$ **and** Σ **has rank** p**, so that** Σ^{-1} **exists, then** $(\underline{X}-\underline{\mu})'\Sigma^{-1}(\underline{X}-\underline{\mu})\sim\chi^2_{(p)}$.

3- Suppose $\underline{X}'=[X_1,X_2,\ldots,X_p]$ **is a** p**-dimensional random variable with mean** $\underline{\mu}$ **and covariance matrix** Σ. **Suppose also that** $Y_j=\underline{a}_j^t\underline{X}$ **is the** j^{th} **principal component, where** $\underline{a}_j$ **is the eigenvector associated with the** j^{th} **largest eigenvalue. Prove that** Y_1 **and** Y_2 **are uncorrelated implies that** $\underline{a}_2^t\underline{a}_1=0$.

4- Using the power method for finding the eigenvalues and eigenvectors, find:

- **i-** **The first principal component of the sample covariance matrix** $S = \begin{bmatrix} .9595 & .7781 \\ .7781 & .7736 \end{bmatrix}$.
- **ii-** **The amount explained by the first principal component.**
- **iii-** **The residual covariance matrix.**
- **iv-** **The correlation coefficients between the first principal component and the variables** X_1 **and** X_2.

5- Prove that

$$-\frac{1}{2}[(\underline{X}-\underline{\mu}^{(1)})'\Sigma^{-1}(\underline{X}-\underline{\mu}^{(1)})-(\underline{X}-\underline{\mu}^{(2)})'\Sigma^{-1}(\underline{X}-\underline{\mu}^{(2)})]$$

$$=\underline{X}'\,\Sigma^{-1}(\underline{\mu}^{(1)}-\underline{\mu}^{(2)})-\frac{1}{2}(\underline{\mu}^{(1)}+\underline{\mu}^{(2)})'\Sigma^{-1}(\underline{\mu}^{(1)}-\underline{\mu}^{(2)})$$

6- Consider two multivariate normal populations with equal covariance matrices, namely $N(\underline{\mu}^{(1)},\Sigma)$ **and** $N(\underline{\mu}^{(2)},\Sigma)$. **Let** $\underline{X}$ **be a random observation, then find the distribution of**

$$U=\underline{X}'\,\Sigma^{-1}(\underline{\mu}^{(1)}-\underline{\mu}^{(2)})-\frac{1}{2}(\underline{\mu}^{(1)}+\underline{\mu}^{(2)})'\Sigma^{-1}(\underline{\mu}^{(1)}-\underline{\mu}^{(2)}).$$

On the assumption that $\underline{X}$ **is distributed according to** $N(\underline{\mu}^{(1)},\Sigma)$.

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Good Luck

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