Cairo University
Institute of Statistical Studies and Research
Department of Applied Statistics and Econometrics

Eco 605: Advanced Econometrics I
Final Exam- May 2015
Time: 3 Hours

Answer the following questions:

Question (1)

1- In simple regression model, $y_i = \alpha + \beta x_i + u_i$, we found that $\sum_{i=1}^{20}(y_i - \bar{y})^2 = 500$, $\sum_{i=1}^{20}(x_i - \bar{x})^2 = 250$, and $\hat{\beta}_{ols} = 1.3$, Find the mean-square error for this model and the correlation coefficient between y and x.

2- Consider the model, $y_t = \alpha + \beta x_t + u_t$, where $u_t = \rho u_{t-1} + \varepsilon_t$; $t = 1,2,\dots$; $|\rho| < 1$, and $E(u_t) = 0$; $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$; $E(\varepsilon_t \varepsilon_s) = 0 \ \forall t \neq s$; $E(\varepsilon_t u_{t-1}) = 0$. Prove that:

(a) $u_t = \sum_{r=0}^{\infty} \rho^r \varepsilon_{t-r}$ (b) $E(u_t) = 0$ (c) $var(u_t) = \sigma_\varepsilon^2/(1-\rho^2)$

3- Assume that $R^2 = 0.952$ for a regression model, and $\sum_{i=2}^{10}(e_i - e_{i-1})^2 = 1.0653$, $\sum_{i=1}^{10}(Y_i - \bar{Y})^2 = 28.744$. where Y_i is the dependent variable, and e_i is the residuals. Test the serial-correlation between the errors using Durbin-Watson test. [note that: $d_L = 0.879$, $d_U = 1.3197$]

Question (2)

Consider the model: $y_i = \beta_0 + \beta_1 i + u_i$, $i = 1,2,3,4,5$

where $E(u_i) = 0$, and $cov(u_i, u_j) = \begin{cases} \sigma^2 i^2 & \forall\ i = j \\ 0 & \forall\ i \neq j \end{cases}$ $i, j = 1,2,3,4,5$

Let $u' = [u_1, u_2, u_3, u_4, u_5]$, and $E(uu') = \sigma^2 V$

(a) Specify V. (b) Find V^{-1}. (c) Find the covariance matrix of the OLS estimator.

(d) Find the covariance matrix of the GLS estimator.

(e) Comment on your results in (c) and (d).

Question (3)

Consider the L-system equations model:

$$\underset{Ln\times 1}{Y} = \underset{Ln\times K}{X}\ \underset{K\times 1}{\beta} + \underset{Ln\times 1}{U}$$

where Y is the vector of endogenous variable, and $X = diag[X_i]$; with X_i $(i = 1,2,\dots,L)$ is the matrix of the exogenous variables of i-equation with dimension $n \times k_i$, and β is the parameters vector with $K = \sum_{i=1}^{L} k_i$, while U is the disturbances vector.

(a) Indicate the assumptions on these equations to we can say that this model is SUR model.

(b) Drive the $\hat{\beta}_{SUR}$ estimator. (c) Prove that $\hat{\beta}_{SUR}$ is BLUE.

(d) Repeat part (b) under an assumption that the all equations in the model are independents. Comment on your results.

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Question (4)

Consider the following model:

$$y_1 = \beta_{11} x_1 + \beta_{12} x_2 + u_1,$$
$$y_2 = \beta_{21} x_3 + \beta_{22} x_4 + u_2,$$

where $cov(u) = cov\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \left[\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \otimes I\right]$. Suppose that data on the dependent and explanatory variables yields:

$y_1'y_1 = 3000$	$y_1'y_2 = 500$	$y_1'x_1 = -200$	$y_1'x_2 = 400$
$y_1'x_3 = 200$	$y_1'x_4 = 100$	$y_2'y_2 = 1000$	$y_2'x_1 = 150$
$y_2'x_2 = -200$	$y_2'x_3 = 30$	$y_2'x_4 = -20$	$x_1'x_1 = 100$
$x_2'x_2 = 300$	$x_3'x_3 = 20$	$x_4'x_4 = 10$	$x_3'x_4 = 10$

$$x_1'x_2 = x_1'x_3 = x_1'x_4 = x_2'x_3 = x_2'x_4 = 0$$

(a) Find the best linear unbiased estimates of $\beta_{11}, \beta_{12}, \beta_{21}$ and β_{22}.

(b) Calculate the SE of these estimates.

(c) Repeat part (a) if $cov(u) = \left[\begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix} \otimes I\right]$.

Question (5)

The model given by:

$$y_{1t} = \gamma_{12} y_{2t} + \beta_{11} x_{1t} + \beta_{12} x_{2t} + u_{1t},$$
$$y_{2t} = \gamma_{21} y_{1t} + \beta_{23} x_{3t} + u_{2t},$$

generates the following matrix of second moments:

	y_1	y_2	x_1	x_2	x_3
y_1	3.5	3	1	1	0
y_2	3	11.5	1	3	4
x_1	1	1	1	0	0
x_2	1	3	0	1	1
x_3	0	4	0	1	2

Calculate the following:

(a) OLS of the reduced form parameters.

(b) ILS estimates of the parameters of the first equation.

(c) 2SLS estimates of the parameters of the second equation.

Good Luck

Dr. Mohamed Reda

Cairo University
Institute of Statistical Studies and Research
Department of Applied Statistics and Econometrics

ECO 605: Advanced Econometrics I
Final Exam- January 2016
Time: 3 Hours

Answer the following questions:

Question (1)

Indicate whether the following statements are **true** or **false**. If a statement is false, rewrite it to make it true.

1- Panel data sets are better able to identify and estimate effects that are simply not detectable in pure cross-sections or pure time-series data.

2- If the errors in a regression model are heteroscedastic, then the estimates and forecasts based on OLS will be unbiased and inconsistent.

3- The main idea of Goldfeld-Quandt test for heteroscedasticity is: if the error variances are equal across observations, then the variance for one part of the sample will be the same as the variance for another part of the sample.

4- White test for heteroscedasticity is a direct test for the heteroscedasticity that is very closely related to the Breusch-Pagan test, but it assumes a prior knowledge of the heteroskedasticity.

5- In seemingly unrelated regression (SUR) equations model, Zellner assumed that the number of equations must to be less than the variables number in the model.

6- The Durbin-Watson (DW) test is valid if the right-hand side of regression equation includes lagged dependent variables.

7- Although White test for heteroscedasticity is a large sample test, it has been found useful in samples of 30 or more.

8- Cochrane-Orcutt iterative procedure requires a transformation of the regression model to a form in which the OLS procedure is applicable.

9- Breusch-Pagan test has been shown to be insensitive to any violation of the normality assumption.

10- If we ignore the positive serial correlation in errors and the independent variable is growing over time, then the standard errors (SE) will underestimate of the true values.

Question (2)

Consider the L-system equations model:

$$\underset{Ln\times 1}{Y} = \underset{Ln\times K}{X}\ \underset{K\times 1}{\beta} + \underset{Ln\times 1}{U} \qquad (1)$$

where Y is the vector of endogenous variable, and $X = diag[X_i]$; with X_i $(i = 1,2,\ldots,L)$ is the matrix of the exogenous variables of i-equation with dimension $n \times k_i$, and β is the parameters vector with $K = \sum_{i=1}^{L} k_i$, while U is the disturbances vector.

(a) Indicate the assumptions on these equations to we can say that this model is SUR model.

(b) Drive the $\hat{\beta}_{SUR}$ estimator. (c) Prove that $\hat{\beta}_{SUR}$ is BLUE.

Question (3)

Consider the following model:

$$y_i = \beta_0 + \beta_1 x_i + u_i;\ i = 1,2,\ldots,6,$$

where $y = (3.531,\ 21.760,\ 49.952,\ 102.214,\ 171.875,\ 243.777)'$, $x_i = 2(i - 0.5)^2$, $E(u_i) = 0$, $E(u_i^2) = \sigma^2 x_i$, and $E(u_i u_j) = 0$ for $i \neq j$.

(a) Estimate β_0 and β_1 by OLS and GLS methods.
(b) Find the variance–covariance matrix of OLS and GLS estimates. Comment on your results.
(c) Find the estimated SE of OLS estimates using White's estimator.
(d) Test the serial correlation between errors using DW test. $[d_L = 0.6102,\ d_U = 1.4002]$

Question (4)

Assume that we can divide the vector of parameters in equation (1) into two vectors based on the prior information that available about β, so the SUR model has been rewritten as:

$$\underset{Ln \times 1}{Y} = \underset{Ln \times k_1}{X_1}\ \underset{k_1 \times 1}{\beta_1} + \underset{Ln \times k_2}{X_2}\ \underset{k_2 \times 1}{\beta_2} + \underset{Ln \times 1}{U}$$

where $E(U) = 0$ and $E(UU') = \sigma^2 V$. Let us assume that strong prior information is available about β_1 vector: $r = \beta_1 + U^*$; where $E(U^*) = 0$, $E(U^* U^{*\prime}) = V_0$, and $E(U^* U') = 0$; where V and V_0 are positive definite matrices.

(a) Use Theil-Goldberger technique to drive the mixed estimator of $\beta = (\beta_1',\ \beta_2')'$.
(b) Prove that the mixed estimator $(\hat{\beta}_M)$ is unbiased.
(c) Prove that $var(\hat{\beta}_M) = \left(\frac{1}{\sigma^2} X'V^{-1}X + R'V_0^{-1}R\right)^{-1}$; where $X = (X_1,\ X_2)$ and $R = (I_{k_1}, 0_{k_2})$.

Question (5)

Assume the following model:

$$y_{1t} = \alpha\, x_{1t} + u_{1t};\quad y_{2t} = \beta\, x_{2t} + u_{2t},$$

where x_{it} $(i = 1,2)$ are non-random exogenous variables and the u_{it} $(i = 1,2)$ are serially independently random variables (disturbances) that are normally distributed with zero means and second moments $E(u_{1t}^2) = 4$, and $E(u_{2t}^2) = E(u_{1t}u_{2t}) = 2$ for all values of t. the sample second moment matrix below was calculated from 20 observations:

(a) Find the best linear unbiased estimates of α and β.
(b) Calculate the SE of these estimates.
(c) Repeat (a) under a restriction that $E(u_{1t}u_{2t}) = 0$.

	y_1	y_2	x_1	x_2
y_1	10	-1	1	-1
y_2	-1	15	5	1
x_1	1	5	11	1
x_2	-1	1	1	1

Good Luck

Dr. Mohamed Reda

Cairo University	Cairo University – Institute Of Statistical Studies And Researches			
	Department: Applied Statistics and Econometrics			
	Academic Year: 2015/2016	Semester: Second		
	Date: 5- 6- 2016	Level: Master		
Course Title: Advanced Econometrics I	**Course code:** EC605	**Time:** 3 Hours	**Exam marks:** 75	**# Exam. Sheets:** 2
Exam. Instructions: Answer the following questions.				

Question One: (15 Marks)

Consider the following model:

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i;\; i = 1,2,\dots,50.$$

We apply the OLS method to estimate the parameters, and then we got these results (*in deviation form*):

$$(x'x)^{-1} = \begin{bmatrix} 0.058 & 0.001 & -0.002 \\ 0.001 & 0.028 & -0.003 \\ -0.002 & -0.003 & 0.020 \end{bmatrix};\; x'y = \begin{bmatrix} 23.169 \\ 169.895 \\ 540.908 \end{bmatrix};\; y'y = 6290.75$$

$$\bar{X}_1 = 1.123;\quad \bar{X}_2 = 1.263;\quad \bar{X}_3 = 1.858;\quad \bar{Y} = 29.250.$$

Depending on these results:

1- Calculate the OLS estimates $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$.
2- Calculate the standard error (SE) of $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$.
3- Calculate the R^2 and the standard error of the model.

Question Two: (20 Marks)

Consider the following model:

$$\underset{n\times 1}{Y} = \underset{n\times k_1}{X_1}\,\underset{k_1\times 1}{\beta_1} + \underset{n\times k_2}{X_2}\,\underset{k_2\times 1}{\beta_2} + \underset{n\times 1}{U}$$

where $E(U) = 0$ and $E(UU') = \sigma^2 V$. Assume that strong prior information is available about β_1 vector: $r = \beta_1 + U^*$, where $E(U^*) = 0$, $E(U^* U^{*\prime}) = V_0$, and $E(U^* U') = 0$, where V and V_0 are positive definite matrices.

1- Use Theil-Goldberger technique to drive the mixed estimator of $\beta = [\beta_1', \beta_2']'$.
2- Prove that the mixed estimator $(\hat{\beta}_M)$ is unbiased.
3- Prove that:

$$var(\hat{\beta}_M) = \left[\frac{1}{\sigma^2} X'V^{-1}X + R'V_0^{-1}R\right]^{-1}, \text{ where } X = [X_1, X_2] \text{ and } R = [I_{k_1}, 0_{k_2}].$$

Question Three: (20 Marks)

Consider the following model:

$$y_i = \beta_0 + \beta_1 x_i + u_i;\quad i = 1,2,\dots,6,$$

where $y = (3.531,\ 21.760,\ 49.952,\ 102.214,\ 171.875,\ 243.777)'$, $x_i = 2(i - 0.5)^2$, $E(u_i) = 0$, $E(u_i^2) = \sigma^2 x_i$, and $E(u_i u_j) = 0$ for $i \neq j$.

1- Estimate β_0 and β_1 by OLS and GLS methods.
2- Find the variance–covariance matrix of OLS and GLS estimates. Comment on your results.

3- Find the estimated SE of OLS estimates using White's estimator.

4- Test the serial correlation between errors using DW test. $[d_L = 0.6102,\ d_U = 1.4002]$.

Question Four: (20 Marks)

Consider the following model: $y_1 = \beta_{11}\, x_1 + \beta_{12}\, x_2 + u_1;\ \ y_2 = \beta_{21}\, x_3 + \beta_{22}\, x_4 + u_2,$

where $cov(u) = cov\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \Sigma \otimes I$. Suppose that data on the dependent and explanatory variables yields:

$y_1'y_1 = 3000$	$y_1'y_2 = 500$	$y_1'x_1 = -200$	$y_1'x_2 = 400$
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$y_2'x_2 = -200$	$y_2'x_3 = 30$	$y_2'x_4 = -20$	$x_1'x_1 = 100$
$x_2'x_2 = 300$	$x_3'x_3 = 20$	$x_4'x_4 = 10$	$x_3'x_4 = 10$
$x_1'x_2 = x_1'x_3 = x_1'x_4 = x_2'x_3 = x_2'x_4 = 0$			

1- If $\Sigma = diag(0.5, 2)$. Calculate the BLU estimates of $\beta_{11}, \beta_{12}, \beta_{21}$ and β_{22}, and the SE of these estimates.

2- Repeat part (1) again but when $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$.

Best Wishes **Prof./: Amany Mousa**

Dr./: Mohamed Reda