Cairo University

Department: Mathematical Statistics

Academic Year: 2016/2017

Academic Semester: Summer Semester

Level: Diploma/ Master/ Ph.D.

Cairo University Institute of Statistical Studies & Research

Subject:	Subject code:	Time:	Exam marks:	# Exam. Sheets:
Mathematical Statistics	MS507	3 Hour	100	2

Exam. Instructions: The examination consists of **seven (5)** questions

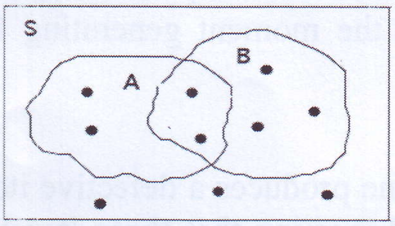
Answer all the following questions

Question 1: 20 marks

I. Prove that for any sample space S of a random experiment, we have

$$P(\phi) = 0.$$

II. The following diagram shows two events A and B in the sample space S. Are the events A and B independent? Explain?



Question 2: 20 marks

I. Determine if each of the following functions is a probability mass function, a probability density function or neither? Explain?

a) $f(x) = \dfrac{x+2}{6}, \quad x = -1,0,1$

b) $f(x) = \dfrac{x-1}{5}, \quad x = 0,1,2,3,4$

c) $f(x) = \dfrac{1}{6}, \quad 0 < x < 6$

d) $f(x) = \dfrac{1}{3}e^{-x}, \quad x > 0$

II. Sixty percent of new drivers have had driving education. During their first year, new drivers with driving education have probability 0.05 of having an accident, while drivers without driving education have a probability 0.08 of having an accident. What is the probability that a new driver has had driving education given that he has had no accident the first year?

Question 3: 20 marks

I. If the probability function of the random variable X is given by

$$f(x) = \begin{cases} p\,(1-p)^x & if\ x = 0,1,2,\dots,\infty \\ 0 & otherwise, \end{cases}$$

Find the expected value and the variance of the random variable X.

II. If the moment generating function of the random variable X is given by

$$M_X(t) = \sum_{x=0}^{\infty} \frac{e^{(tx-1)}}{x!}$$

What is the probability of the event $X = 2$.

Question 4:20 marks

I. If X is a Binomial random variable with parameters p and n, find with proof the mean, the variance and the moment generating function of the random variable X.

II. The probability that a machine produces a defective item is 0.02. Each item is checked as it is produced. Assuming that these items are independent trials, what is the probability that at least 50 items must be checked to find one that is defective?

Question 5: 20 marks

I. Prove that if X is uniform on the interval [a, b], then the mean, variance, and moment generating function of X are given by

$$a)\ E(x) = \frac{a+b}{2}, \quad b)\ Var(x) = \frac{(b-a)^2}{12}, \quad c)\ M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}, \quad t \neq 0$$

II. If X is a gamma random variable with parameters $\alpha = 1$ and $\theta = 1$, then what is the probability that X is between its median and its mean.

Best Wishes **Dr./:Mohamed Abdelhamid Sabry**