Cairo University

Institute of Statistical Studies and Research

Second Semester 2011-2012

Pre-Master Students

Stochastic Processes in Demography (Stat M 602)

Time: Three hours

Answer the Following Questions

Q1. Define the following: Periodic state - Ergodic chains – Absorbing state - Row stochastic matrix - null Persistent state.

Q2. For the m- state Markov chain, let T be an $m \times m$ stochastic matrix. c is eigenvectors matrix, and D is the diagonal matrix of eigenvalues. Use the diagonalization method to prove that $T^n = CD^nC^{-1}$.

Q3. A three stat Markov chain has the transition matrix

$$T = \begin{pmatrix} p & 1-p & 0 \\ 0 & 0 & 1 \\ 1-q & 0 & q \end{pmatrix}, \quad 0 < p,q < 1$$

Show that:

a) the state E_1 is persistant.

b) the probability that the system returns at least once to E_1 is equal to 1.

Q4. Prove that: $P^{(n+r)} = P^{(r)}T^n$.

Q5. Discuss if states of the following chain are ergodic?

$$T = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Q6. Discuss if the following two state chain is

$$T = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

a) Irreducible.

b) Positive matrix.

Q7. In the following Markov chain

$$T = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{2}{7} & \frac{5}{7} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

a) sketch its transition diagram.

b) Which states do you think are transient?

c) Which states do you think are closed irreducible sub-chain?